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PERSONAL EQUATION AND STEADINESS OF JUDGMENT IN THE ESTIMATION OF THE NUMBER OF OBJECTS IN MODERATELY LARGE SAMPLES

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Investigations in experimental evolution, involving as they necessarily do the recording of the characteristics of large numbers of individuals, afford the opportunity of obtaining quite incidentally extensive series of data upon errors of observation or estimation. The present paper states the major results of one such series of records obtained at the Station for Experimental Evolution during the course of investigation requiring the counting of large numbers of beans for mass weighings, germination tests, etc.

The chief advantages of such observations lie in the facts that they are carried out under quite natural working conditions, with none of the artificiality of the laboratory test, and that they represent far larger experiments than the average professional psychologist is able to make. Comprising as they do 28 experiments due to three observers all of whom carried on the work at considerably separated intervals over a period of two years, during which they made over 15,000 estimates with determined errors, the constants have a reliability which cannot possibly be attributed to short series.

The routine of this work was so organized that it consisted in part of a series of attempts to lay out samples of a definite number (25, 50, 100, or 200) which was constant for considerable periods. The error of each estimate was at once determined and recorded.

Two characteristics of the series of errors of estimation made by the three observers are here considered—personal equation and steadiness of judgment.

By personal equation we understand a definite bias in a given direction. In a series of estimates by an observer the errors in excess of the true value may be no more numerous and no greater in amount than those in defect. The average of the deviation of the estimates from the true number of objects will then be 0 plus or minus a small amount due to the errors of random sampling. Such an observer may be said to have no personal equation. Other individuals, however, may have a definite tendency to err on one side of verity in their evaluations. Such may be said to have a positive or a negative personal equation, as the case may be. Personal equation is measured by the mean, regarding signs, of the deviation of the samples from their ideal value.

But quite without reference to personal equation, one observer may be more erratic than another, estimating now far too high, now far too low. By steadiness of judgment we mean consistency in estimation as measured by the closeness with which the errors of estimation cluster around their mean value. Steadiness of judgment may be expressed in the absolute terms of the standard deviation of the errors of estimation about their mean (S.D.), or in the relative terms of the coefficient of variation (C.V.).

$$S.D. = \sqrt{\frac{\text{Sum of (Deviations from Mean)}^2}{\text{Total estimates}}}$$

which here is most easily calculated from the formula

$$(S.D.)^2 = \Sigma(d^2)/N - [\Sigma(d)/N]^2,$$

where Σ is the conventional summation sign, N is the number of estimates and d indicates the deviation of the estimate from the true number of objects, *i. e.*, the actual number laid out less the required number; and

$$C.V. = 100\, S.D./M,$$

where M is the constant number which the observer seeks to lay out plus or minus the observed personal equation, as the sign of the latter may indicate.

In much of the work in which personal equation is a factor the observer is not able to check his estimates against the true values, and so attempt at each successive observation to profit by his previous experience. In these experiments each observer made a persistent effort to improve. This was based on a knowledge of the immediately preceding errors, and consisted in a constant effort to lay out exactly the desired number of seeds. Thus the influence of experience upon both

personal equation and steadiness of judgment may be determined from these data.

The problems here taken under consideration fall, therefore, into two groups. *First*, those having to do with the existence of personal equation in the estimation of the number of objects in samples and of differences in personal equation and steadiness of judgment from individual to individual. *Second*, the influence of previous experience upon personal equation and steadiness of judgment.

In the case of all three observers there is a slight but significant personal equation, which, notwithstanding the constant effort to improve, persisted throughout the two years during which the experiments were intermittently made. *In only three out of the twenty-eight experiments did the observer lay out samples of too small average size.* In a large number of the individual experiments the personal equation is certainly statistically significant (trustworthy) in comparison with its probable error.

From the experimental data taken as a whole one cannot conclude that there is any demonstrated difference between the personal equation of the three observers, although the figures do suggest that the bias of observer *D* may be slightly greater than that of either of the others. All have a bias in the direction of laying out more than the intended number of seeds, but that one is worse than another cannot be asserted.

In a high proportion of the individual experiments the differences between the personal equations of the three observers are statistically significant in relation to their probable errors. This is true in cases in which (for example) *B* has a greater personal equation than *C*, as well as in these in which she has a smaller personal equation.

The probable explanation of this result seems to be that the observers vary somewhat in their personal equation from experiment to experiment, just as they vary from time to time in general health, physiological tone, and mental vigor, alertness, or whatever one may care to call it. As a result of this variation from time to time one observer may show an abnormally high personal equation in a particular experiment in which a second observer shows an unusually low one. On another occasion the condition may be exactly reversed.

Thus *in an individual experiment* one observer may seem to be decidedly better than another. *In the long run* there is no *fully demonstrated* difference between them.

For steadiness of judgment there is no absolute standard comparable with the zero mean deviation of the personal equation. The data show a coefficient of variation of about 6.9% in the case of Observer *B* and *C*,

and of 8.7% in the case of Observer *D*, who has a decidedly greater scatter in her estimates—that is a far less steady judgment—than either of the other observers. Indeed, in every individual experiment her standard deviation is higher than that of either of the two other experimenters.

Thus while there is no certain differentiation among the experimenters in personal equation, they differ distinctly in steadiness of judgment.

The influence of previous experience upon personal equation or steadiness of judgment may be most succinctly expressed in terms of the correlation between some quantitative measure of the amount of previous experience and the measures of personal equation and steadiness of judgment.

In these experiments the errors of observation were recorded in sequence. A group of fifty consecutive estimates with the accompanying determinations of the errors constituted a 'period.' In determining correlations one must deal with a number of subgroups for each period. It is most convenient to divide each half daily period of 50 estimates into five consecutive 'trials,' each of 10 estimates. For each of these 'trials' the mean personal equation and the standard deviation of the errors must be computed. Thus in obtaining the constants discussed here it was first necessary to compute 1520 means and 1520 standard deviations, which were then treated as units in computing the correlations.

The main problems involved in the question of experience are two: Is there a change in personal bias as a result of constant effort to improve and opportunity for improvement? Does the judgment become steadier, *i.e.*, does the observer make less erratic estimates, as a result of experience?

Both of these questions are really twofold. Is there an improvement from period to period? Is there an improvement within the period? In short, does the worker improve both from estimate to estimate in the same half daily period and also from period to period?

Personal equation seems to be remarkably little influenced by experience. In some experiments it increases, in others it decreases. The correlations may be either positive or negative in sign. Numerically they are generally low, and are in great part insignificant in comparison with their probable errors. Taken as a whole the results indicate a slight reduction in personal equation as a result of experience from period to period. Within the period there is no demonstrable influence of experience upon personal equation.

Steadiness of judgment is in rather conspicuous contrast with per-

sonal equation, in that it is unmistakably influenced by previous experience. The correlations between the number of previous trials within the period and steadiness of judgment and between the number of previous periods of experience and steadiness of judgment are numerically low, but almost without exception indicate that as experience becomes greater the scatter of the individual estimates about their mean value becomes less. Probably the rate of this change is not uniform, but is most rapid at first and then falls off.

The full data and discussion are appearing in two papers in the *Psychological Review*.

POLYPEPTIDE-HYDANTOINS

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Carbon dioxide is one of the products of decomposition when certain proteins undergo hydrolysis, under normal conditions, by digestion with aqueous solutions of acids and alkalis. Mörner[1] observed the formation of this acid anhydride during an investigation of the action of hydrochloric acid (sp. gr. 1.124) on horn at 92°, but no quantitative determination of the gas was made and no special significance attributed by him to its formation. Lippich[2] confirmed this observation several years later and showed that this anhydride is a normal product of hydrolysis of other proteins. He also made the important observation that the quantity actually formed is dependent on the nature of the hydrolytic agent employed. Quantitative determinations of the amount of the gas evolved from several proteins under specific conditions revealed the interesting fact that the maximum quantity is obtained when an alkali, as potassium or barium hydroxide, is used as the hydrolytic agent. In no case did Lippich fail to detect the presence of this substance among his products of hydrolysis. The actual percentages obtained by hydrolysis of five different proteins with potassium hydroxide solution are recorded in Table I.

For his acid hydrolyses Lippich used 33% sulphuric acid. When these same proteins were broken down by heating with this reagent entirely different analytical results were obtained. The combinations in the proteins productive of carbon dioxide were more resistant to hydrolytic changes, under these conditions, and the maximum amount of this gas was not obtained until after 25–27 hours digestion. The percentages found are recorded in Table I.